General Relativity

Cosmologists generally use some concepts and equations of general relativity as the most appropriate approach to describe space and to get models fitting the observations and the big bang picture. General relativity has been tested in various ways. One test was to measure the precession of the perihelion of the planet Mercury. Another was to measure the bending of starlight for stars seen near the edge of the sun when it was eclipsed. Another was to measure the redshift of light when leaving a star with a very strong gravitational field. General relativity theory passed these tests. More recently, with more advanced equipment, tests have been made by bouncing laser beams from the earth off retroreflectors on the moon. Results were also consistent with general relativity.

The delay and deflection of light signals passing close to massive objects have now been measured with increasing precision and are in agreement with the predictions of general relativity at the 0.1% level. Geodetic precession has been detected by the laser ranging to the moon coupled with radiointerferometry data. Gravitational radiation from accelerated masses in a binary pulsar system has been shown to be consistent with general relativity at the 0.4% level.

There are still untested aspects of general relativity and there are still competing theories. In orbit is a NASA experiment called Gravity Probe B to test general relativity predictions for an effect called frame dragging (to 1%) and geodetic effects (to a part in 10,000) and to see how other theories fare. The concepts and equations of general relativity are used in the development of most cosmological models favored by cosmologists, however, and these concepts are used in what follows.

One such concept of general relativity is that we should not think of galaxies rushing away from us through a fixed space, we should think of galaxies more or less standing still in their local space but that space itself is expanding. This is a difficult concept but it can be described mathematically. In this concept, light leaving a distant galaxy is traveling at velocity c toward us in that local space but since space itself is expanding and that local space is moving away from us, the velocity of a light packet toward us (defined in a certain way) is less than c until the

packet gets to our local space and the velocity can even be negative for some early portion of the time light travels to us.
Cosmologists also use a principle called the Cosmological Principle which says that if we look at sufficiently large regions of space, space is homogeneous and isotropic, the same everywhere and the same in all directions, there is no center. This means that the rate of expansion of space can be expressed in terms of a single scale factor R(t) which is the same at all points of space at any one time but will be a function of time for an expanding or contracting universe.
The proper distance (explained below) between any two galaxies as a function of time is proportional to R(t) (neglecting small local motions of the galaxies as we will do in most of what follows).
Another concept has to do with the apparent luminosity of distant galaxies. If we define the absolute luminosity, L_A, as the total radiant power from a galaxy over all angles and define the apparent luminosity, L_a, as the radiant power reaching us per unit area of our telescope, then in the simple case of a nearby galaxy, $L_a=L_A/4pd^2$ where d is the distance to that galaxy, hence $d=(L_A/4pL_a)^{1/2}$. This is called the luminosity distance, D_l. (There may have to be an additional correction, called the K correction, for the shape of the energy spectrum of the emitting galaxy and the limitations of the spectral range of the measuring instruments which measure L_a.) (L_A is based on observations of nearby galaxies and assumes a fairly consistent total radiant power of even the distant galaxies of similar type.)
Since for more distant galaxies the light has been traveling toward us for a long time, the question arises, is this the proper distance away the galaxy was when it emitted the light we see now, or the proper distance away it is now or even something else? With this concept of general relativity of space itself expanding, reflection will show that the correct distance to use is the proper distance **away now** when the light is received, D_r.
There is also a need to make a correction for the wavelength of the received photons as the light has been stretched out as the universe has expanded during its travel. This reduces the energy by the factor of 1/(z+1). Also, the rate at which the photons are being received is reduced by the same factor so L_a is reduced by $1/(z+1)^2$. The final equation then for D_r is:

$$D_r = D_l/(z+1).\text{--------------}(2)$$

This distance is called the proper distance or proper comoving distance. See Narlikar, *Introduction to Cosmology* pp 94-97**(1)** for this derivation.

Now we have several concepts of distance. To understand the proper distance let's use a simple analogy. Suppose our universe is only two-dimensional and is modeled by the surface of a balloon which is being steadily inflated. A sticker on the balloon represents our position and other stickers represent the positions of other galaxies. At a particular instant let's measure with a tape along the surface of the balloon the distance from our sticker to another sticker representing the galaxy under observation. This measurement is the proper distance. Another way to describe it is to imagine starting at our position in space in the real universe with a series of observers in a straight line to the galaxy being examined, each carrying a non-expanding measuring stick and measuring the distance to his neighbor at a particular instant. The sum of these measurements is the proper distance at that instant. This seems like the most physically meaningful distance concept. So now, we have the luminosity distance, D_l, the proper distance now, D_r, and the proper distance then, D_e.

There is one more concept of distance, the light travel distance. Imagine an insect crawling along the surface of the balloon from the distant sticker to our sticker. If the insect counted its number of steps to get here and knew the length of each step and multiplied these numbers, it would come up with a distance greater than the distance then but less than the distance now. Call this distance D_{lt} or the light-travel distance. Thus $D_e < D_{lt} < D_r$. Thus, $D_{lt} = ct_{lb}$. Note that we now have four concepts of distance. The differences are illustrated in Figure IV related to the third model to be discussed.

An excellent paper on these distance concepts is "Another Look at Cosmic Distances" by Thomas A. Weil in Sky and Telescope for August 2001. The author explains the different distance concepts and give formulas but also supplies a computer code for calculating the different distances and explains the effects of matter density and the cosmological constant (explained below).

One can also think of the analogy of a muffin filled with raisins. Before the muffin is baked, the raisins are close together. As it is baked and the yeast causes it to rise, the raisins all move away from each other, and the further apart two raisins are at the beginning the more rapidly they move away from each other. The distance apart of two raisins is the proper distance in this analogy when this separation is measured with a ruler.

The balloon analogy is helpful in another way. The balloon surface is a closed 2-dimensional curved surface of finite area, yet it has no edges. One can imagine it starting from almost a point then inflating to a certain size, stopping and contracting again. Another surface would be a flat surface of infinite extent. Still another would be something like a saddle-shaped surface, also of infinite extent.

These illustrate in 2 dimensions what some of the concepts of general relativity are of curved 3-dimensional surfaces which we cannot readily visualize but which can be described mathematically. The balloon analogy carried to a higher dimension is called a hypersphere.

For the cosmological models most used by cosmologists, (neglecting something called the cosmological constant which will be explained later), if the present mass density of the universe is large enough, space is positively curved (like the balloon). If the mass density is a certain critical value, space is flat. For lower mass densities, space is negatively curved, something like saddle-shaped. (In a space of positive curvature the sum of the interior angles of a triangle will be greater than 180^0, for flat space, the sum of the angles will equal 180^0, and for negatively curved space the sum will be less than 180^0. You would need an enormously large triangle to see this effect.) For a positively curved (or closed) universe, gravity is strong enough that the expansion will slow down, stop and reverse. The volume of this universe will be finite, and so will its lifetime. For the universe of just the critical density, the density will remain critical (see note at the end of this section) and the expansion will slow down and only stop after an infinite time (dR/dt-->0). Such a universe is generally considered to be open or infinite in extent and to be flat. At lesser densities the universe (also open) will expand forever. For a universe of this present lower mass density, if we look back in time the density approaches the critical density closer and closer to be virtually identical to it just after the big bang. For a description of these spaces see *The Shape of Space* **(2)** by Jeffrey Weeks.

The balloon analogy helps us to think about questions like, if space is flat or saddle-shaped, does space have an edge? Is there more of the universe we cannot now see? Are there more universes different from ours? These are difficult questions but the balloon analogy helps us to see that for at least some universe models space can be finite in extent but still not have an edge. There is a recent article in the Scientific American**(3)** discussing these questions of the extent and shape of space and suggesting possibilities for space to be flat or saddle-shaped but still finite in what is referred to as a multiply-connected universe. In the next few years, more observations of galaxy distributions may shed light on this issue. Some cosmologists think that model universes which have zero or negative curvature and yet are finite in extent are "pathological" while others want to search for evidence that our universe might be of such a kind. Recent data further support the view that space is not positively curved but flat See the May 27, 2004 update in Latest News.

Another concept of general relativity is that the red shift of light from distant galaxies due solely to **the expansion of space itself** is not given by the relativistic Doppler shift equation of special relativity but is simply related to the present scale factor of the universe compared to the scale factor when the light was emitted that we see now. The equation is:

$$z+1=R(t_r)/R(t_e)\text{-----------}(3)$$

The relativistic Doppler effect is appropriate for the redshifts due to local motions such as the rotation of our own Milky Way galaxy and its motion toward a nearby galaxy cluster, the Virgo cluster.

The equation for the total redshift when both the cosmological redshift and Doppler redshift are involved is given in Peebles *Principles of Physical Cosmology*(4), pages 96-98.

While in special relativity there is no standard of rest and no preferred reference frame, in general relativity the situation is somewhat different. A rest frame for **any particular region** of space can be defined as the frame which is not rotating compared to the background of distant stars and for which the cosmic microwave background radiation, CMBR, has the same spectrum in all directions and so the concept of local motion in that frame is meaningful. In another region of space, there would also be a rest frame for which the CMBR is the same in all directions, but the two rest frames are moving with respect to each other so there is no preferred frame of that type for the universe as a whole and no center of the universe. We can however, define a **comoving** rest frame for the universe as a whole for which the requirement of the CMBR being the same in all directions and there being a standard of rotation based on the distant stars is met at all locations.

Still another concept is that, since we are thinking of space itself expanding rather than of galaxies rushing through space, and since we are assuming that galaxies are nearly standing still in their local spaces for these simple models, we need not use the equations of special relativity of the difference in clock rates of observers in uniform motion with respect to each other, we can think, instead, of a single universal time for the universe as a whole.

General relativity also allows for the introduction of something called the cosmological constant. For now, assume that the cosmological constant is not involved in the first few models to be discussed.

Note on critical mass density. In the Einstein deSitter model (our Model III) space is flat, there is no cosmological constant (dark energy) and mass density is critical. That critical density means that the universe will expand forever but will slow down closer and closer to stopping after an infinite time. But if the density is critical now and if the density in **kilograms per cubic meter** obviously goes down as the universe expands, how can it always remain critical? We must look at the definition of critical mass density in the equations. It is:

$r=3H^2/8pG$ . . . . hence it depends on H, the Hubble constant.

r, The density in kilograms per cubic meter, varies as $1/a^3$ as the universe expands where a is the scale factor of the universe, normalized to 1 at the present time. How does H, the Hubble "constant" vary with time? It is a constant over space at any time but does decrease continually with time in this cosmological model.

H is defined in the equation for receding galaxies as v=Hs or H=v/s. (v is velocity and s is distance.) Assume a galaxy away from us a distance s, and the present value of s is s_0. At another time the distance is s_0a. The recession velocity then is s_0da/dt and $H=(s_0/s)da/dt=(1/a)da/dt$. For this Einstein deSitter model, $a\sim t^{2/3}$ so $da/dt\sim t^{-1/3}$ or $da/dt\sim 1/a^{1/2}$ and $H\sim 1/a^{3/2}$ and $H^2=1/a^3$. It was noted above that r varies in exactly the same way with a as $1/a^3$ and so if the density is once critical it is always critical in this model. QED.

This situation is different for the model (our Model V) in which there is a positive value for the cosmological constant. Such a universe could start out with very rapid expansion slowing down as in our Model III but then beginning to accelerate and finally accelerating exponentially forever. Thus it would not gradually slow down to a stop. In this case the question of density has more to do with the shape of space, flat or otherwise.

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