

# DISTANT GALAXIES AND COSMOLOGICAL MODELS

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## APPENDIX I

When a very distant galaxy or quasar is observed there are several different quantities measured or calculated or previously given. These might include the following quantities (very few of which are usually mentioned in articles).

### Measured:

1.  $L_a$  - the apparent luminosity (corrected for redshift as needed) (The flux received)
2.  $z$  - the cosmological or relativistic Doppler red shift
3.  $N(z)_{\text{obs}}$  - Observed galaxy count out to redshift  $z$  **Given or assumed:**
4.  $L_A$  - the absolute luminosity assumed for the object (galaxy or quasar or supernova) (The total power emitted over all angles)
5.  $d$  - the linear diameter assumed for the galaxy
6.  $R(t)$  - the scale factor of the universe (depending on the model)
7.  $r_0$  - the present mass density of the universe
8.  $\Lambda$  - the cosmological constant
9.  $G$  - the gravitational constant **Calculated:**
10.  $D_l$  - the luminosity distance of the object  $(L_A/4\pi L_a)^{1/2}$
11.  $t_e$  - the age of the universe when the light was emitted that we see now
12.  $t_r$  - the age when the light is received (Also  $t_0$  for the present age)
13.  $D_e$  - the proper distance away the object was at time  $t_e$
14.  $D_r$  - the proper distance away the object is at  $t_r$  or  $t_0$
15.  $D_{\text{light}}$  - The proper distance away from us now of a light packet emitted from **our** galaxy at time  $t_e$
16.  $D_{\text{lt}}$  - the light "travel distance",  $ct_{\text{lb}}$
17.  $v_e$  - the velocity of recession (rate of change of proper distance)

- when the light was emitted we see now
18.  $v_r$  - the velocity of recession (rate of change of proper distance) when the light is received (now)
  19.  $t_{lb}$  - the "look back" time, i.e., the time the light has been traveling
  20.  $H_0$  - the Hubble "constant" at the present time
  21.  $q$  - the angular diameter of the galaxy (could also be measured)
  22.  $r$  - the mass density of the universe (could also be assumed)
  23.  $D_{ph}$  - the particle horizon
  24.  $D_{eh}$  - the event horizon
  25.  $dN/dz$  - the relative number of galaxies counted in a telescope field of view as a function of distance increment expressed in terms of  $z$ .
  26.  $n$  - the number of new galaxies coming into view each year
  27.  $W_m$  - the mass density of the universe divided by the critical mass density (Could be assumed)
  28.  $W_l$  - the cosmological constant,  $\Lambda$ , divided by  $3H_0^2$  (or assumed)
  29.  $W_k$  - the space curvature term,  $k$ , divided by  $-H_0^2$  (or assumed)

## APPENDIX II Einstein deSitter cosmological model equations

For this model the scale factor varies as  $R(t) \sim t^{2/3}$ , the density is critical, space is flat and  $t_0 = 2/3H_0$ . In a scientific paper, these features are usually described by saying  $k=0$  (space is flat),  $q_0 = 1/2$  (the universe is decelerating),  $W_m = 1$  (the universe has the critical mass density) and  $W_l = 0$  (there is no cosmological constant).

(The subscript  $_0$  denotes values at the present time for the quantities listed.) Hence:

$$R(t_0)/R(t) = (t_0/t)^{2/3} = 1 + z \text{ -----(1)}$$

The proper comoving distance of a galaxy from us now is:

$$D_r = 3ct_0^{2/3}(t_0^{1/3} - t_e^{1/3}) \text{ -----(2)}$$

( $t_e$  is the time when the light was emitted from it that we see now and  $t_0$  is  $t_r$ .)

This can be written as:

$$D_r = (2c/H_0)(1 - 1/(1+z)^{1/2}) \text{ -----(2a)}$$

Suppose we think of the question the other way: how far away

from us now is a light packet emitted from our galaxy at time  $t_e$ ? We can see that  $D_{\text{light}}=D_r$  and hence equations (2) and (2a) are correct for  $D_{\text{light}}$ .

The proper comoving distance away the galaxy was when it emitted the light we see now is:

$$D_e = D_r / (1+z) = 3ct_e^{2/3} (t_0^{1/3} - t_e^{1/3}) \text{ -----(3)}$$

$$D_e = (2c/H_0) (1 - 1/(1+z)^{1/2}) / (1+z) \text{ -----(3a)}$$

This is, of course, the proper distance away from us the light packet was when it left the galaxy so we can use this equation to trace the distance away from us a light packet was from very very distant galaxies all the way back to nearly the big bang when  $t_e \sim 0$  and  $z$  approaches infinity.

For this model, the particle horizon is at a proper distance of:

$$D_{\text{ph}} = 3ct_0 \text{ -----(4)}$$

*Note: This is not  $ct_0$  as so many books state. (Maybe they use the light travel distance,  $ct_{\text{lb}}$ ?)*

There is no event horizon for this model.----- (4a)

There is a distance concept called the luminosity distance,  $D_l$ . If the total luminosity of a galaxy is  $L_A$  over all angles and if the observed luminosity per unit area is  $L_a$ , then the luminosity distance is defined as:

$$D_l = (L_A / 4\pi L_a)^{1/2} \text{ -----(5)}$$

and the present proper comoving distance for this galaxy is:

$$D_r = D_l / (1+z) \text{ -----(5a)}$$

The mass density for this model is the critical density:

$$\rho = 3H^2 / 8\pi G \text{ -----(6)}$$

As the particle horizon recedes, new galaxies are coming into view each year. The number of such galaxies per year,  $n$ , is:

$$n = N_0 / t_0 \text{ -----(7)}$$

If there are  $10^{11}$  galaxies now and  $t_0 = 15$  billion years, then  $n = 7$ . The observed angular diameter of a galaxy of linear diameter  $d$  is:

$$q = d / D_e = (dH_0 / 2c) (1+z) / (1 - (1+z)^{-1/2}) \text{ -----(8)}$$

This can also be expressed in terms of the present distance of

the galaxy,  $D_r$  as:

$$q = (d/D_r) / (1 - D_r/3ct_0)^2 \text{ ----- (8a)}$$

Interestingly, this angle decreases with increasing distance at first as would be expected, but then reaches a minimum at  $z=1.25$  and  $D_r=ct_0$  and gradually increases for very distant galaxies and large  $z$ 's.

The look-back time,  $t_r - t_e$  is given by:

$$t_{lb} = t_0(1 - 1/(1+z)^{3/2}) \text{ ----- (9)}$$

**A concept of recession velocity** can be introduced as  $v = dD/dt$  and so  $v_0 = dD_r/dt$  and  $v_e = dD_e/dt$ . (These velocities will be different in this model because the universe is slowing down).

$$v_0 = 2c(1 - (1+z)^{-1/2}) \text{ ----- (10)}$$

$$v_e = 2c((1+z)^{1/2} - 1) \text{ ----- (10a)}$$

*Note: The recession velocity **defined in this way** can clearly exceed  $c$  which is consistent with some texts, but many texts and articles say that the velocity can never exceed  $c$ . For this definition of velocity, velocities greater than  $c$  are possible for galaxies we can still see.*

The Hubble constant,  $H_0$ , is defined as  $v = H_0 d$ , but for this Einstein deSitter model, which  $v$  is it,  $v_e$  or  $v_0$ , and which  $d$  is it,  $D_e$  or  $D_r$ ? In terms of present velocity,  $v_0$ , the results are:

$$D_e = (v_0/H_0)(1 - v_0/2c)^2 \text{ ----- (11)}$$

$$D_r = v_0/H_0 \text{ (surprise!) ----- (11a)}$$

For a homogeneous, isotropic universe the galaxy count per increment of  $z$  as function of  $z$  is:

$$dN/dz \sim (1 - (1+z)^{-1/2}) / (1+z)^{3/2} \text{ ----- (12)}$$

As a function of  $D_r$ , however it is  $dN/dD_r \sim D_r^2 \text{ ----- (12a)}$

### Appendix III Equations for Models II through V

NOTE: If you don't have the "Symbol" font, you won't be able to view some of the Greek letters correctly.

If this is the case, email us, and we will fax or mail you a written

copy.

For models II through V of this paper, if the scale factor of the universe,  $R(t)$  is known, the value of  $H(t)$  can be found as well as the trajectory in the time-distance plane of receding galaxies (neglecting local motions) and of the light packet reaching us now from these galaxies. The behavior of the mass density over time and the impact of the cosmological constant can also be determined. From general relativity an equation called the Friedmann equation has been developed which permits  $R(t)$  to be determined. The equation is:

$$\left(\frac{da}{dt}\right)^2 = H_0^2 \left( \frac{W_{m0}}{a} + W_{i0} a^2 + W_{k0} \right) \text{-----(1)}$$

Where:

$a = R(t)/R(t_0)$  (The scale factor normalized to 1 at the present time.)

$W_{m0}$  = present mass density/critical mass density

$(8\pi G \rho_0 / 3H_0^2)$   $W_{i0}$  = the cosmological constant term at the present time  $(\Lambda / 3H_0^2)$   $\Lambda$  is the cosmological constant itself  $W_{k0}$  = the space curvature term at the present time  $(-k/H_0^2)$   $k$  is negative for negatively curved space, 0 for flat space and positive for positively curved space

From these definitions it can be seen that  $W_{m0} + W_{i0} + W_{k0} = 1$ . Thus  $a(t)$  can be found by step-by-step integration of equation (1).

From this,  $H(t)$  can be found and the space-time trajectory of the light package reaching us now can also be found. This is how the curves for Figure V and VI were generated.

The values of  $W_{m0}$ ,  $W_{i0}$ , and  $W_{k0}$  for Models II through V are:

For Model II, 0, 0, 1

For Model III, 1, 0, 0

For Model IV, 0.3, 0, 0.7

For Model V, 0.3, 0.7, 0

$W_m(a)$  - the mass density at any time divided by the critical

mass density is given by the equation:

$$W_m(a) = (W_{m0}/a) / (W_{m0}/a + W_{\Omega_0 a^2} + W_{k0}) \quad (2)$$

When this is evaluated for a given value of  $a$ , it can be determined as a function of time since  $a(t)$  is known.

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## Graphs

[Fig I](#) Model I Paths (moving through space) and redshifts [Fig II](#)  
Model II Paths (steadily expanding space) and redshifts [Fig III](#)  
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