APPENDIX IV

THE TWIN PARADOX

A paradox is a seemingly contradictory statement that may nonetheless be true. The twin paradox in special relativity goes like this: There are twins living together. Let's call them A and B. After setting their clocks each to 0, twin A stays home and twin B goes at high speed, v, on a journey to a distant point at a distance S away and turns around and comes back home. When they compare clocks, they find that twin B's clock shows less elapsed time, he is now younger. Special relativity says that the rate of a clock depends on the speed, v, with which it is moving compared to the reference frame and the equation is t'=t times sqrt(1-v^2/c^2). In this first case the reference frame is the one in which twin A stays at rest. The paradox comes about because it would seem that if we took as our reference frame the frame in which twin B is at rest on the outbound leg and twin A is thus moving throughout the adventure, then twin A's clock should be slower hence the seeming contradiction.

We should realize however that in the reference frame of twin B at the start he is not moving but after the distant point passes him he has to go at much greater speed than the other twin's speed in order to catch up and hence his clock is more greatly slowed. The equations for this way of looking at the problem are more complicated but reduce to the same thing in the end and there is no contradiction.

The equations go as follows: on the outbound leg twin B is at rest and twin A is moving at velocity v (for the whole adventure). B sees the distance S shortened to be S times sqrt(1-v^2/c^2). When the distant point itself passes B, his clock will show S/v times sqrt(1-v^2/c^2). B must now go at high speed to catch up to A, call it v'. It would at first seem that B's speed would be 2v, however in special relativity speeds do not simply add. The equation is v'=2v/(1+v^2/c^2). In a chase the time to catch up is the distance divided by the **difference** in speeds, hence the time to catch up is given by the equation

$$t = \frac{S\sqrt{1-v^2/c^2}}{\frac{2v}{1+v^2/c^2}-v} = \frac{S}{v}\frac{\sqrt{1-v^2/c^2}}{(1-v^2/c^2)}\cdot\left(1+\frac{v^2}{c^2}\right) = \frac{S}{v}\frac{(1+v^2/c^2)}{\sqrt{1-v^2/c^2}}$$

However, B's clock is running very slowly because of this high catch-up speed. The equation for this correction is: t'=t times sqrt(1=v^2/c^2).

Substituting for v' we get:

$$t' = t\sqrt{1 - \frac{4v^2}{c^2(1-v^2/c^2)^2}} = t\,\frac{(1-v^2/c^2)}{(1+v^2/c^2)}$$

Applying this correction to the time involved in the catch-up we get:

$$t = \frac{S}{v}\,\frac{(1+v^2/c^2)}{\sqrt{1-v^2/c^2}} \times \frac{(1-v^2/c^2)}{(1+v^2/c^2)} = \frac{S}{v}\sqrt{1-v^2/c^2}$$

And the total time shown on B's clock at catch-up is:

$$\frac{S}{v}\sqrt{1-v^2/c^2} + \frac{S}{v}\sqrt{1-v^2/c^2} = \frac{2S}{v}\sqrt{1-v^2/c^2}$$

And this is in agreement with the result we had when we took the reference frame where A is at rest for the whole adventure.

Now, what about A's clock as viewed in this reference frame?

For the first leg it would be just $\frac{S}{v}\sqrt{1-v^2/c^2}$

And fro the second leg it would be: $\frac{S}{v}\,\frac{(1+v^2/c^2)}{\sqrt{1-v^2/c^2}}$

Adding these we get $\frac{2S}{v} \times \frac{1}{\sqrt{1-v^2/c^2}}$ but A's clock is running slowly in this reference frame by the factor $\sqrt{1-v^2/c^2}$ so the final answer is 2 S/v just as in the first case, so there is no contradiction and special relativity has passed this test of the paradox.

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